

# “Finishing” an Open Textbook

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Joint Mathematics Meeting

Seattle, WA

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After teaching the course in Spring 2015, I had:

- 10 course note packets (125 pages)
- 21 in-class activities
- 12 homework sets (45 questions)
- 9 practice problem sets (150 questions)

In time for Fall 2015:

- 4 main chapters (200 pages)
- *Investigate!* activities introducing most sections.
- Homework problems at end of each chapter
- 300 practice problems.

# Organization of Content

What is a chapter/section/subsection?

Where should the practice problems go?

Where should the homework problems and activities go?

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# Layout choices

- Real textbooks have light blue boxes for definitions and theorems!
- Examples and Activities should stand out as well.
- LaTeX package: mdframed.
- In text, use

```
\begin{definition}... \end{definition}
```

then fix layout later.

```
\renewenvironment{definition}[1]{%
  \mdfsetup{%
    frametitle={\colorbox{blue!20}{\space#1\space}},
    frametitlealignment={\hspace*{1ex}},
    frametitleaboveskip=-1.5ex,
    frametitlebelowskip=0pt,
    roundcorner=1pt,
    leftmargin=3pt,
    rightmargin=3pt,
    backgroundcolor=blue!5,
    linecolor=blue!75!black,
  }
  \begin{mdframed}%
}{%
  \end{mdframed}%
}
```

## Chapter 3

# How to Count

One of the first things you learn in mathematics is how to count. Now we are going to learn that all over again, and you will find that counting is a lot harder than you remember. The problem is that we want to count large collections of things quickly and precisely. For example:

- How many different lotto tickets are possible?
- In a group of 10 people, if everyone shakes hands with everyone else exactly once, how many handshakes took place?<sup>1</sup>
- How many ways can you distribute 10 girl scout cookies to 7 boy scouts?
- How many 10 digit numbers contain exactly 4 prime digits?
- How many functions  $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3\}$  are onto?
- How many subsets of  $\{1, 2, 3, \dots, 10\}$  have cardinality 7?

Before tackling these difficult questions, let's look at the basics of counting.

### 3.1 Additive and Multiplicative Principles

Consider this rather simple counting problem: at Red Dogs and Donuts, there are 14 varieties of donuts, and 16 types of hot-dogs. If you want either a donut or a dog, how many options do you have? This is an easy question - you just add 14 and 16. Will that always work? What is important here?

#### Additive Principle

The *additive principle* states that if an event  $A$  can occur in  $m$  ways, and event  $B$  can occur in  $n$  disjoint ways, then the event " $A$  or  $B$ " can occur in  $m + n$  ways.

It is important that the events be disjoint. For example, a standard deck of 52 cards contains 26 red cards and 12 face cards. However, the number of ways to select a card which is either red or a face card is not  $26 + 12 = 38$ . This is because there are 6 cards which are both red and face cards.

<sup>1</sup>We know the answer to this thanks to graph theory - we are asking how many edges there are in  $K_{10}$ . What if your group had a secretive 3-person handshake? If every group of three people participated in such a handshake, how many handshakes take place? Equivalently, how many triangles are there in  $K_{10}$ ?

The additive principle works with more than two events. Say you would also consider eating one of 15 waffles? How many choices do you have now? You would have  $14 + 16 + 15 = 45$  options.

**Example:** How many two letter “words” start with either A or B? How many start with one of the 5 vowels? (A word is just a strings of letters - they don’t have to be English words, or even pronounceable).

*Solution:* First, how many two letter words start with A? We just need to select the second letter, which can be accomplished in 26 ways. So there are 26 words starting with A. There are also 26 words that start with B. So to select a word which starts with either A or B, we can pick the word from the first 26 or the second 26, for a total of 52 words. The additive principle is at work here.

Now what about all the two letter words starting with a vowel? Well there are 26 starting with A, another 26 starting with E, and so on. We will have 5 groups of 26. So we add 26 to itself 5 times. Of course it would be easier to just multiply  $5 \cdot 26$  - we are really using the additive principle again, just using multiplication as a shortcut.

**Example:** Suppose you are going for some FroYo - you can pick one of 6 yogurt choices, and one of 4 toppings. How many choices do you have?

*Solution:* Break your choices up into disjoint events:  $A$  are the choices with the first topping,  $B$  the choices featuring the second topping, and so on. So we have events. Each can occur in 6 ways (one for each yogurt flavor). The events are disjoint, so the total number of choices is  $6 + 6 + 6 + 6$ .

Note that in both of the previous examples, when using the additive principle on a bunch of sets all the same size, it is quicker to multiply. This really is the same - not just because  $6 + 6 + 6 + 6 = 4 \cdot 6$ . We can first select the topping in 4 ways (that is we first select which of the disjoint events we will take). For each of those first 4 choices, we now have 6 choices of yogurt. We have:

#### Multiplicative Principle

The *multiplicative principle* states that if event  $A$  can occur in  $m$  ways, and each possibility for  $A$  allows for exactly  $n$  ways for event  $B$ , then the event “ $A$  and  $B$ ” can occur in  $m \cdot n$  ways.

The product rule generalizes to more than two events as well.

**Example:** How many license plates can you make out of three letters followed by three numerical digits?

*Solution:* Here we have six events: the first letter, the second letter, the third letter, the first digit, the second digit and the third digit. The first three events

## COUNTING

One of the first things you learn in mathematics is how to count. Now we want to count large collections of things quickly and precisely. For example:

- In a group of 10 people, if everyone shakes hands with everyone else exactly once, how many handshakes took place?
- How many ways can you distribute 10 girl scout cookies to 7 boy scouts?
- How many anagrams are there of “anagram”?
- How many subsets of  $\{1, 2, 3, \dots, 10\}$  have cardinality 7?

Before tackling these difficult questions, let’s look at the basics of counting.

**1.1 ADDITIVE AND MULTIPLICATIVE PRINCIPLES****Investigate!**

1. A restaurant offers 8 appetizers and 14 entrées. How many choices do you have if:
  - (a) you will eat one dish, either an appetizer or an entrée?
  - (b) you are extra hungry and want to eat both an appetizer and an entrée?
2. Think about the methods you used to solve the counting problems above. Write down the rules for these methods.
3. Do your rules work? A standard deck of playing cards has 26 red cards and 12 face cards.
  - (a) How many ways can you select a card which is either red or a face card?
  - (b) How many ways can you select a card which is both red and a face card?
  - (c) How many ways can you select two cards so that the first one is red and the second one is a face card?



**Do not proceed until you have attempted the activity above**



Consider this rather simple counting problem: at Red Dogs and Donuts, there are 14 varieties of donuts, and 16 types of hot dogs. If you want either a donut or a dog, how many options do you have? This isn't too hard, you just add 14 and 16. Will that always work? What is important here?

#### Additive Principle

The *additive principle* states that if event  $A$  can occur in  $m$  ways, and event  $B$  can occur in  $n$  disjoint ways, then the event " $A$  or  $B$ " can occur in  $m + n$  ways.

It is important that the events be disjoint. For example, a standard deck of 52 cards contains 26 red cards and 12 face cards. However, the number of ways to select a card which is either red or a face card is not  $26 + 12 = 38$ . This is because there are 6 cards which are both red and face cards.

The additive principle works with more than two events. Say, in addition to your 14 choices for donuts and 16 for dogs, you would also consider eating one of 15 waffles? How many choices do you have now? You would have  $14 + 16 + 15 = 45$  options.

#### Example:

How many two letter "words" start with either A or B? How many start with one of the 5 vowels? (A word is just a strings of letters; it doesn't have to be English, or even pronounceable.)

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**Solution:** Break your choices up into disjoint events:  $A$  are the choices with the first topping,  $B$  the choices featuring the second topping, and so on. So we have four events. Each can occur in 6 ways (one for each yogurt flavor). The events are disjoint, so the total number of choices is  $6 + 6 + 6 + 6$ .

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And an index.

Perhaps a list of symbols.

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What should the first page of the pdf look like?

What about the cover of the physical book?



# DISCRETE MATHEMATICS

AN OPEN INTRODUCTION

OSCAR LEVIN

FALL 2015

# DISCRETE MATHEMATICS



AN OPEN INTRODUCTION

OSCAR LEVIN

FALL 2015

# Other design considerations

- Font (newpx with some customization).
- Pagination.
- Headings.
- Page numbers.

## Other technical considerations

- File structure (package: docmute)
- Organize problems and solutions (package: answers)
- Custom environments that will play nice with exam doc-class.
- Homework problems don't have solutions but practice problems do.
- Graphics: TiKZ or . . . .

# Licensing and Copyright

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- Reviews???

How often should a new edition come out? How should the editions be numbered?

- 1 Improve the book (fix typos, more activities, etc.)
- 2 More distribution formats.
- 3 WeBWork problems.
- 4 ???
- 5 Not profit.

# Looking forward

How often should a new edition come out? How should the editions be numbered?

- 1 Improve the book (fix typos, more activities, etc.)
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# Thanks

Slides and Textbook Info:



`discretetext.oscarlevin.com/talks.php`