Tricks to Make Counting Harder for Students

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An example

How many anagrams of "anagram" are there?

$$\binom{7}{3}4!$$

or

$$\frac{7!}{3!}$$

Why counting is hard

- No way to check solutions
- Multiple ways to write solutions
- Every problem is a word problem
- Differences in problem types are subtle
- Others reasons?

How to help students

Option 1: make counting easier

Option 2: make counting harder

Option 1

How many ways... ...n labeled boxes?	at most one per box	any number per box	exactly one per box
k labeled (ordered) balls	**A:** $\binom{n}{k}k! = n(n-1)\ldots(n-k+1)$	**E, F:** (k_j balls unordered within box) $\frac{k!}{k_1!k_2!\ldots k_n!} = \binom{k}{k_1}\binom{k-k_1}{k_2}\binom{k-k_1-k_2}{k_3}\ldots\binom{k_n+k_{n-1}}{k_{n-1}}$	———
k unlabeled (unordered) balls	**B:** $\binom{n}{k}$	**D, D':** $\binom{k+n-1}{k} = \binom{k+n-1}{n-1}$ and $\binom{k-1}{n-1} = \binom{k-1}{k-n}$	———
unlimited balls, k different labels (order matters)	———	———	**C:** k^n

from *Discrete Mathematics with Ducks*, sarah-marie belcastro, CRC Press 2012

Key Words

When do you add? When do you multiply?

Combinations vs permutations ($C(n,r)$ vs $P(n,r)$).

"**Problem-Solving Tips**: The key points to remember in this section are that a permutation takes order into account and a combination does *not* take order into account. Thus, a key to solving counting problems is to determine whether we are counting ordered or unordered items."

Discrete Mathematics by Richard Johnsonbaugh, 7th ed. Pearson, 2009

Spelling is also hard

I before E except after C...

...or in one of the following 3529 exceptions:

beige, cleidoic, codeine, conscience, deify,
deity, deign, dreidel, eider, eight, either,
feign, feint, feisty,foreign, forfeit, freight,
gleization, gneiss, greige, greisen, heifer, heigh-ho,
height, heinous, heir, heist, leitmotiv, neigh,
neighbor, neither, peignoir, prescient,
rein, science, seiche, seidel, seine,
seismic, seize, sheik, society, sovereign,
surfeit, teiid, veil, vein, weight,

Option 2

- We cannot ignore these rules.
- Most students know them.
- Creating rules, making generalizations, etc. is an important mathematical skill.
- But we can use the rules as a teaching tool.

and/or

Selecting from 6 soups and 5 salads:

- How many ways can you pick a soup **or** a salad?
- How many ways can you pick a soup **and** a salad?
- How many ways can you pick one menu item from the list of soups **and** salads?

Selecting from a standard deck of 52 playing cards:

- How many ways can you pick a red card **or** a face card?
- How many ways can you pick a red **and** face card?

Does order matter?

- How many ways can you arrange four 0’s and six 1’s?
- How many 4 digit numbers have their digits in decreasing order?
- How many 5-letter “words” can you make if (a) the letters can go in any order, or (b) the letters must be in alphabetical order.

Option 2.5: Mathematics to the Rescue

To really help our students, we must make counting harder in another way.

1. Require mathematically rigorous explanations from students.
2. Shift focus to understanding mathematical models for counting.

Set theory

Every counting question:

How many elements are in this set?

Sorts of sets to count:

- Sets of elements.
- Sets of tuples.
- Sets of subsets (bit strings).
- Sets of multisets.
- Sets of functions.
- Sets of injective functions.

How could you represent elements in the set?

Add or multiply?

How many lattice paths from $(0,0)$ *to* $(6,6)$ *pass through* $(2,3)$*?*

$$\binom{5}{2} + \cdot \binom{7}{4}$$

Should you add or multiply?

Are you taking the union or Cartesian product of two sets?

Combination or Permutation

At your bookstore you want to display 4 of the 10 NYTimes bestsellers on the top shelf. How many ways can you do this if any order (left to right) for the books is acceptable?

How could you represent elements in the set?

$$\{A, C, D, G\}$$

$$\{C, A, G, D\}$$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ A & C & D & G \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ C & A & G & D \end{pmatrix}$$

Equivalence relations?

Counting permutations means counting injective functions.

Counting combinations is counting injective functions, modulo some equivalence relation (permutations of the domain)

Too much?

Or a perfect preview for Lagrange's Theorem?

Making Counting Impossible (on purpose)

How many ways can you select 7 jelly beans out of 20 flavors?

Thanks!

Slides and free textbook:

discretetext.oscarlevin.com/talks.php